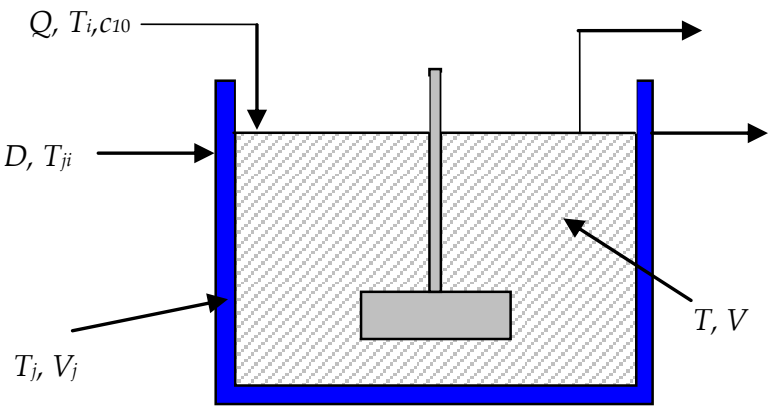
Exercise: Process identification using ARX and ARMAX models

Let us consider a perfectly mixed chemical reactor (CSTR) where an irreversible first order chemical reaction takes place: A → B.



The mass and heat balance for the systems are described by the following equations:

$$\frac{dT}{dt} = -\frac{UA}{\rho VC}\left(T - T_j\right) + \frac{Q}{V}\left(T_i - T\right) - \frac{\Delta H_1}{\rho C} k_{0,1} e^{-\frac{E_1}{RT}} c_1$$

$$\frac{dc_1}{dt} = -k_{0,1} e^{-\frac{E_1}{RT}} c_1 + \frac{Q}{V}\left(c_{1,0} - c_1\right)$$

$$\frac{dT_j}{dt} = \frac{UA}{\rho_j V_j C_j}\left(T - T_j\right) + \frac{D}{V_j}\left(T_{j,i} - T_j\right)$$

U = 2160 kJ m^{-2} h^{-1}	ρ = 1000 kg m^{-3}
A = 0.2 m^{-2}	ρ_j = 1000 kg m^{-3}
$k_{0,1}$ = 5616 m^{3}mol^{-1}h^{-1}	C = 4.18 kJ kg^{-1} K^{-1}
E_1 = 31.4 kJ mol^{-1}	C_j = 4.18 kJ kg^{-1} K^{-1}
ΔH_1 = -87.4 kJ mol^{-1}	V = 9·10^{-3} m^3
D = 1.4·10^{-4} m^3 h^{-1}	V_j = 4·10^{-3} m^3

The initial conditions are:

$T(t=0)$ = 368 K

$c_1(t=0)$ = 5 mol m^{-3}

$T_j(t=0)$ = 368 K

The values of the input variables are:

$c_{1,0}$ = 100 mol m^{-3}

T_i = 293 K

$T_{j,i}$ = 372 K

Q/V = 8.6 ·10^{-4} min

1. Assuming that the output variable of interest is the **concentration of the reactant**, and that the manipulated variable is the **temperature of the fluid in the jacket**, determine if the system is linear or not.
2. Determine the **time constant** of the process, and create a random sequence of input values for the identification procedure. In this framework it is possible to assume a variation of the input value of ±30% around the initial value.
3. Create the set of input-output data required by the process identification algorithm.
4. Check if the ARX model is suitable for the identification of the process.
5. Determine the parameters of the ARMAX model that approximates the dynamics of the system.
6. Study the performance of the model in case the input variable is increased of 20%.