

4.3 Dynamic Real-Time Optimization

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Outline

Introduction

Overview of DRT0 approaches

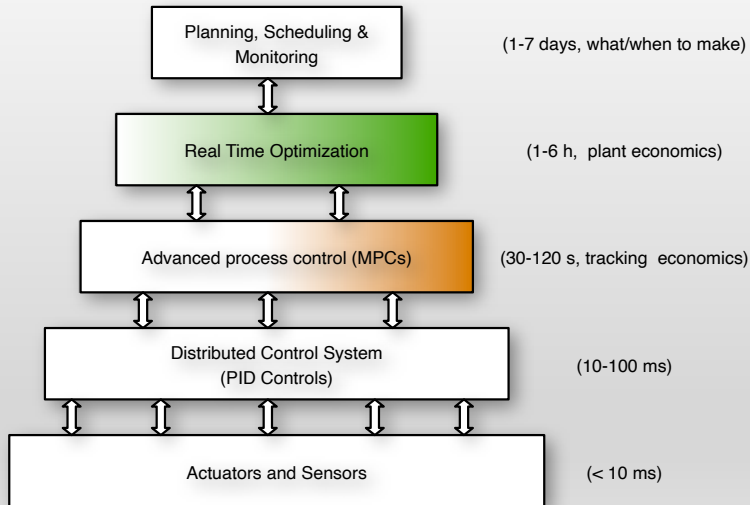
Economic NonLinear Model Predictive Control

Software tool for NMPC

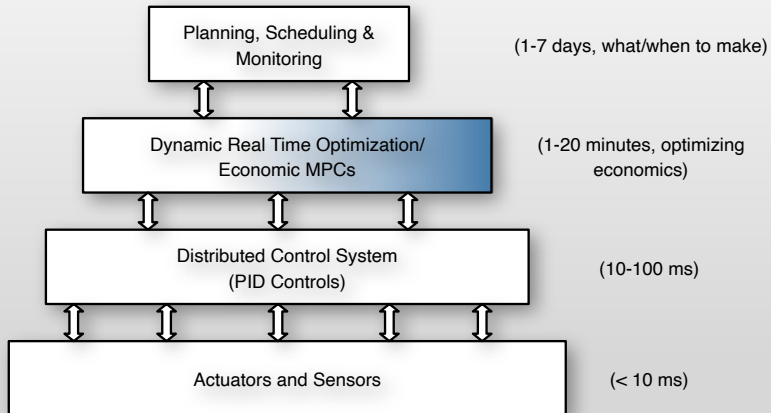
An industrial example

Conclusions

Hierarchical scheme of optimization, monitoring and control



Revised hierarchical architecture



Dynamic real-time optimization

Motivations and goals

Motivations

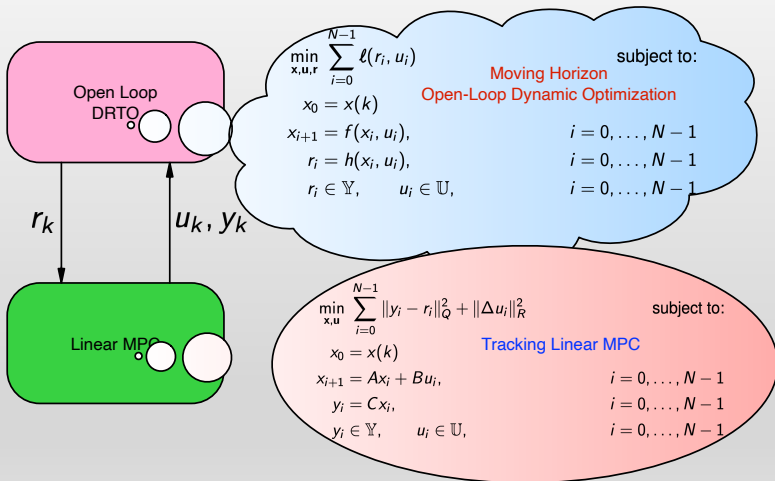
- ▶ Process plants are operating in an increasingly global and dynamic environment
- ▶ Account for transient behavior in the determination of economically optimal operating policies
- ▶ Inconsistency between the RTO and control layers (offset-free strategies needed)

Goals

- ▶ Optimize plant economics dynamically, taking into account changes in plant parameters (e.g. raw materials, energy price, throughput, etc.)
- ▶ Meet operating constraints

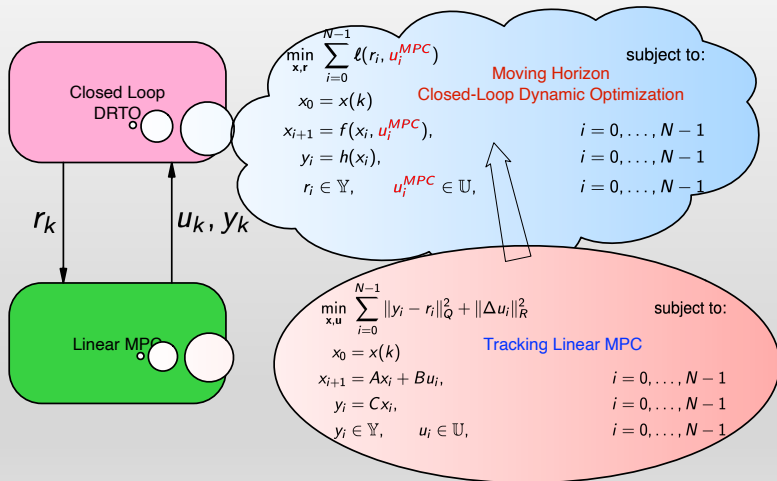
Two-layer approaches

Open-loop DRTO / MPC

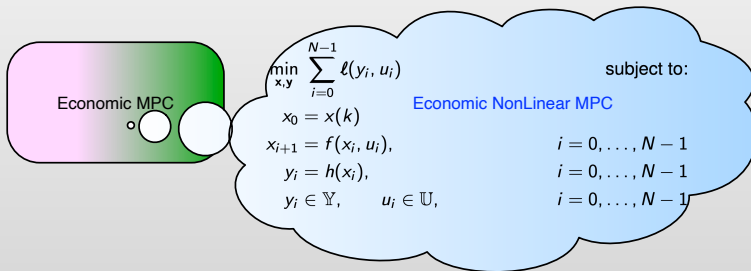


Two-layer approaches

Closed-loop DRTO / MPC



Single-layer approaches



NMPC formulation for reference tracking

Nominal formulation

Finite-horizon optimal control problem

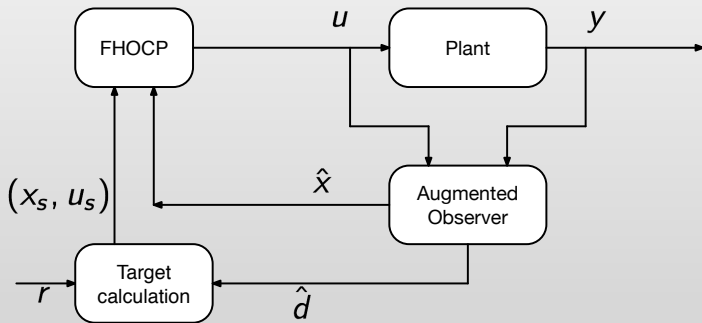
$$\begin{aligned} \min_{\mathbf{x}, \mathbf{u}} \quad & \|x_N - x_s\|_P^2 + \sum_{i=0}^{N-1} \|y_i - x_s\|_Q^2 + \|u_i - u_s\|_R^2 & \text{subject to:} \\ & x_0 = x(k) \\ & x_{i+1} = f(x_i, u_i), \quad y_i = h(x_i) & i = 0, \dots, N-1 \\ & y_i \in \mathbb{Y}, \quad u_i \in \mathbb{U}, & i = 0, \dots, N-1 \end{aligned}$$

Equilibrium target

$$\begin{aligned} \min_{x_s, u_s, y_s} \quad & \|y_s - r\|_T^2 & \text{subject to:} \\ & x_s = f(x_s, u_s), \quad y_s = h(x_s) \\ & y_s \in \mathbb{Y}, \quad u_s \in \mathbb{U} \end{aligned}$$

NMPC formulation for reference tracking

Offset-free formulation: block diagram



NMPC formulation for reference tracking

Offset-free formulation: augmented model

Nominal and augmented model

► **Nominal** model:

$$x^+ = f(x, u)$$

$$y = h(x)$$

► **Augmented** model:

$$x^+ = F(x, d, u)$$

$$d^+ = d$$

$$y = H(x, d)$$

with **consistent dynamics**: $F(x, 0, u) = f(x, u)$ and $H(x, 0) = h(x)$

NMPC formulation for reference tracking

Offset-free formulation: target calculation

Equilibrium target

$$\min_{x_s, u_s, y_s} \|y_s - r\|_T^2$$

subject to:

$$x_s = F(x_s, \hat{d}_{k|k}, u_s), \quad y_s = H(x_s, \hat{d}_{k|k})$$

$$y_s \in \mathbb{Y}, \quad u_s \in \mathbb{U}$$

NMPC formulation for reference tracking

Offset-free formulation: FHOCP

Finite-horizon optimal control problem

$$\min_{\mathbf{x}, \mathbf{u}} \|\mathbf{x}_N - \mathbf{x}_s\|_P^2 + \sum_{i=0}^{N-1} \|H(\mathbf{x}_i, \hat{\mathbf{d}}_{k|k}) - \mathbf{y}_s\|_Q^2 + \|\mathbf{u}_i - \mathbf{u}_s\|_R^2$$

subject to:

$$\mathbf{x}_0 = \hat{\mathbf{x}}_{k|k}$$

$$\mathbf{x}_{i+1} = F(\mathbf{x}_i, \hat{\mathbf{d}}_{k|k}, \mathbf{u}_i),$$

$$i = 0, \dots, N-1$$

$$H(\mathbf{x}_i, \hat{\mathbf{d}}_{k|k}) \in \mathbb{Y}, \quad \mathbf{u}_i \in \mathbb{U},$$

$$i = 0, \dots, N-1$$

Economic MPC

Standard formulation

FHOCP where $\ell(\cdot)$ is **economic cost function**

$$\min_{\mathbf{x}, \mathbf{u}} \sum_{i=0}^{N-1} \ell(H(\mathbf{x}_i), \mathbf{u}_i) \quad \text{subject to:}$$

$$\mathbf{x}_0 = \mathbf{x}(k)$$

$$\mathbf{x}_{i+1} = f(\mathbf{x}_i, \mathbf{u}_i), \quad i = 0, \dots, N-1$$

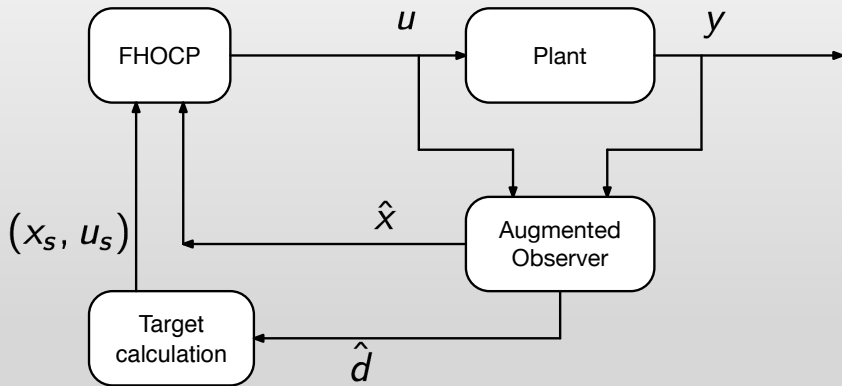
$$H(\mathbf{x}_i) \in \mathbb{Y}, \quad \mathbf{u}_i \in \mathbb{U}, \quad i = 0, \dots, N-1$$

About the cost function

- $\ell(\cdot)$ measures the process economics during transient, not deviation from setpoints, which don't exist

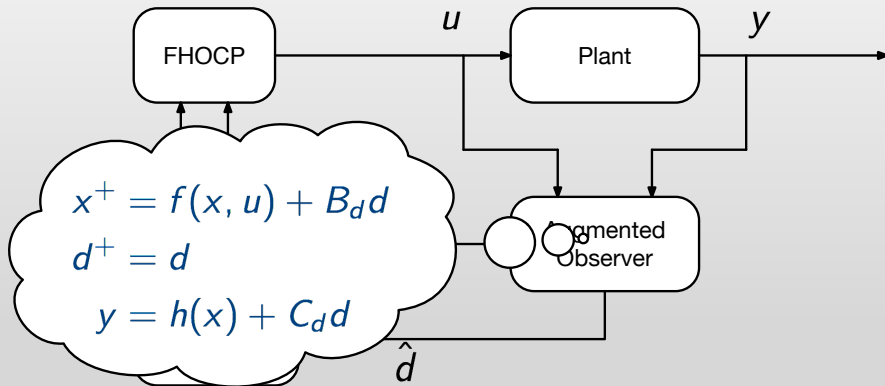
Economic MPC

Offset-free formulation



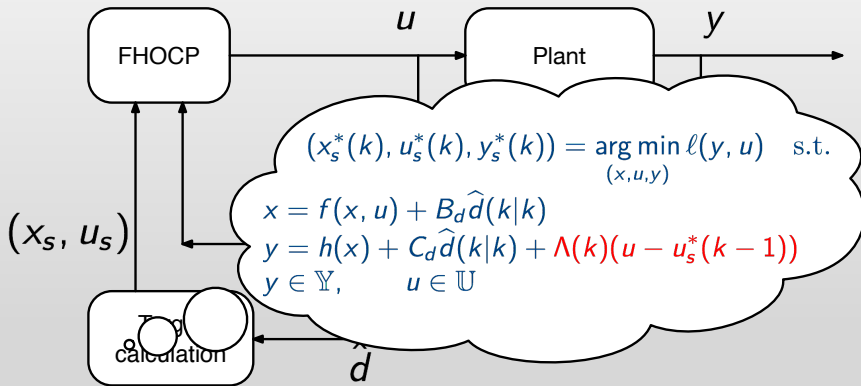
Economic MPC

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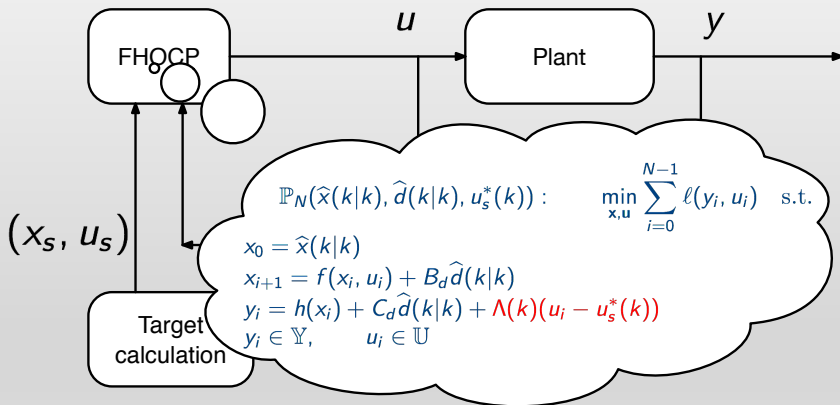
Economic MPC

Offset-free formulation



Economic MPC

Offset-free formulation



Open-source software tool for NMPC

Presentation

<https://github.com/CPCLAB-UNIFI/MPC-code>

Welcome to the MPC-code!

We present a multipurpose, easy-to-use code for Model Predictive Control (MPC) design, analysis and simulation. The major goal of this code is to provide the user with a general, versatile MPC framework that can be adapted to problems in different areas...

- ▶ **Python** Amply validated, fast, easy-to-use, open-source, customization.
- ▶ **CasADi** Open-source symbolic calculation through algorithmic differentiation, numeric optimization oriented.
- ▶ **IPOPT** Standard in the class of open-source nonlinear programming (NLP) solvers.



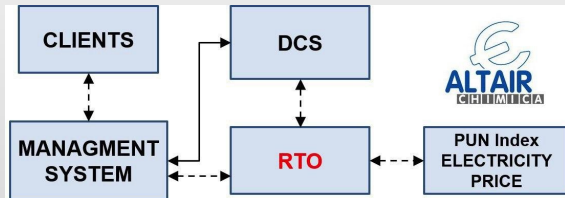
CasADi



Industrial case study

Problem definition: project scheme

Enhancing the factory management of **Altair Chimica SPA** with:
automation, digitalization, machine learning and process computerization



Goals

Develop a **RTO system** to model and **optimally schedule** the production plan

- ▶ exchange input and output data with the DCS at fixed times
- ▶ hierarchically superior to controllers: works as a **fully automatic operator**

Project major components

- ▶ **Management System**: handle the orders from clients and make the sale plan by hand
- ▶ Distributed Control System (**DCS**)



Industrial case study

Problem definition: Nomenclature

n_p products. Each j -th has

- ▶ Production rate: $x_j = [x_j^0, \dots, x_j^i, \dots, x_j^{n_h-1}]$
- ▶ Sales plan: $S_j = [S_j^0, \dots, S_j^i, \dots, S_j^{n_h-1}]$

n_h are the hours in the **optimization horizon**, i.e. $n_h = 24 \times 7 = 168$ h in a week.

Sales plan features

- ▶ From the selling department and used within the problem as **input parameters**
- ▶ Defining $t_{s,d}$, with $d = 1, \dots, 7$ as the selling time of each day
 - ▶ the only non-zero components of S_j are the ones for $i = t_{s,d}$
 - ▶ sale is satisfied iff the stock of product j contains enough material at time $t_{s,d}$

Batch vs Continuous products

- ▶ $n_b < n_p$ products are produced with batch reactors
- ▶ The corresponding x_j is zero throughout most of the horizon

Industrial case study

Problem definition: Nomenclature

Storable vs Non-Storable products

Storable

- ▶ Stock of product j is **function of** x_j and S_j
- ▶ Stock is **bounded by physical constraints**
- ▶ **Mass balance** from the initial stock σ_j^0 :

$$\sigma_j^{i+1} = \sigma_j^i + x_j^i - S_j^i - a_j(x)^i \quad \forall i = 0, \dots, n_h$$

Non-Storable

- ▶ Some products cannot be stocked due to specific **safety or logistic reasons**
- ▶ **Cannot be stocked or sold** \rightarrow must be consumed within the facility
- ▶ **Mass balance** collapse to:

$$0 = x_j^i - a_j(x)^i \quad \forall i = 0, \dots, n_h$$

Self-consumption $a_j(x)$

Some of the products are **consumed** within the industrial site **to obtain other chemicals**



Industrial case study

Batch Scheduling: the methodology through an example

The sales plan

	$t_{s,1}$	$t_{s,2}$	$t_{s,3}$	$t_{s,4}$	$t_{s,5}$	$t_{s,6}$	$t_{s,7}$
P_1	0	$S_{P_1}^{t_{s,2}}$	0	$S_{P_1}^{t_{s,4}}$	0	0	0
P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Example

- 7-day plan
- 3 batch products and 3 reactors
- All initial stocks empty

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
R_1							
R_2							
R_3							

$t_{s,2}$



Industrial case study

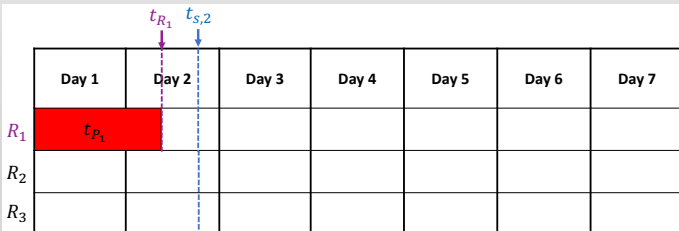
Batch Scheduling: the methodology through an example

The sales plan

	$t_{s,1}$	$t_{s,2}$	$t_{s,3}$	$t_{s,4}$	$t_{s,5}$	$t_{s,6}$	$t_{s,7}$
P_1	0	$S_{P_1}^{t_{s,2}}$	0	$S_{P_1}^{t_{s,4}}$	0	0	0
P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- Consider $S_{P_1}^{t_{s,2}} > W_P$
- 1 batch reaction for P_1 in R_1
- Reactor time $t_{R_1} < t_{s,2} \rightarrow \checkmark$





Industrial case study

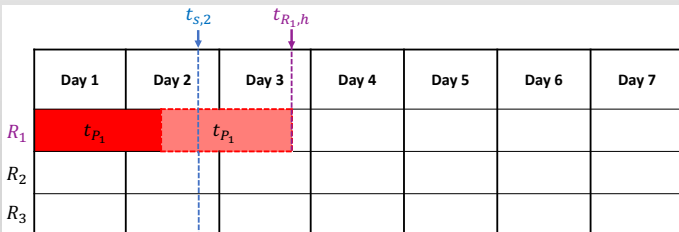
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The sales plan

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P_1	0	$S_{P_1}^{t_{s,2}}$	0	$S_{P_1}^{t_{s,4}}$	0	0	0
P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- ▶ $S_{P_1}^{t_{s,2}} > W_P \rightarrow$ 2° batch of P_1 has to be scheduled
- ▶ 2° batch reaction for P_1 in R_1
- ▶ Tentative reactor time $t_{R_1,h} > t_{s,2} \rightarrow \text{X}$





Industrial case study

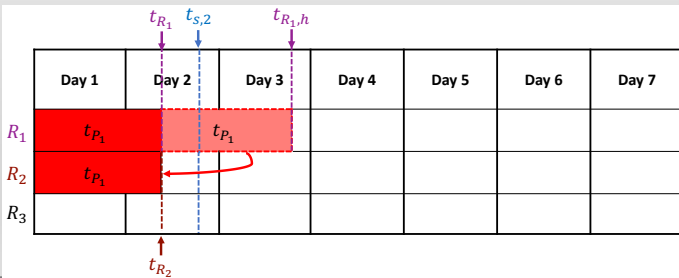
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The sales plan

	$t_{s,1}$	$t_{s,2}$	$t_{s,3}$	$t_{s,4}$	$t_{s,5}$	$t_{s,6}$	$t_{s,7}$
P_1	0	$S_{P_1}^{t_{s,2}}$	0	$S_{P_1}^{t_{s,4}}$	0	0	0
P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- R_2 is enrolled
- 1 batch reaction for P_1 in R_2
- Reactor time $t_{R_2} < t_{s,2} \rightarrow \checkmark$





Industrial case study

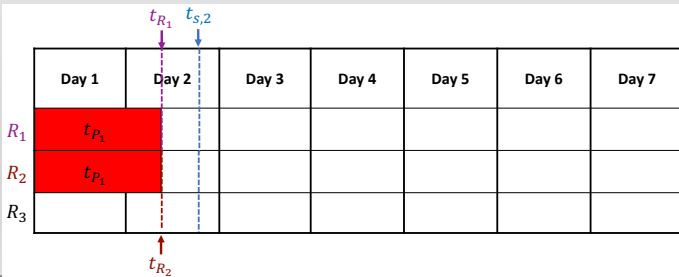
Batch Scheduling: the methodology through an example

The sales plan

	$t_{s,1}$	$t_{s,2}$	$t_{s,3}$	$t_{s,4}$	$t_{s,5}$	$t_{s,6}$	$t_{s,7}$
P_1	0	$S_{P_1}^{t_{s,2}}$	0	$S_{P_1}^{t_{s,4}}$	0	0	0
P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- The first sale is satisfied correctly
- $t_{R_1} = t_{P_1}$, $t_{R_2} = t_{P_1}$, $t_{R_3} = 0$



Industrial case study

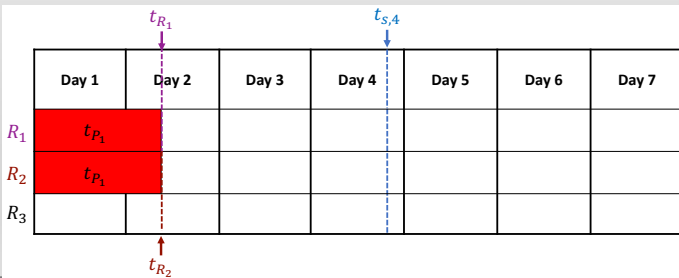
Batch Scheduling: the methodology through an example

The sales plan

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P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- ▶ $S_{P_2}^{t_{s,4}} > W_P$ and $S_{P_1}^{t_{s,4}} \leq W_P$
- ▶ 2 batch for P_2 and 1 batch for P_1 required
- ▶ P_2 has the priority





Industrial case study

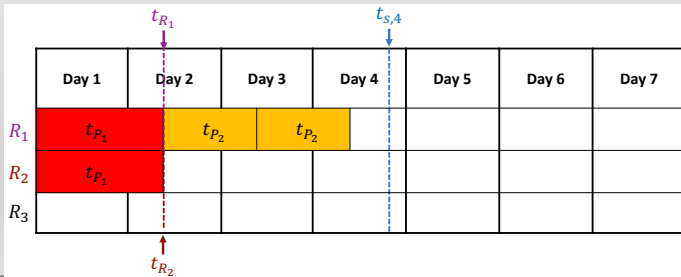
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P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- ▶ 2 batch reaction for P_2 in R_1
- ▶ Reactor time $t_{R_1} + 2 * t_{P_2} < t_{s,4} \rightarrow \checkmark$





Industrial case study

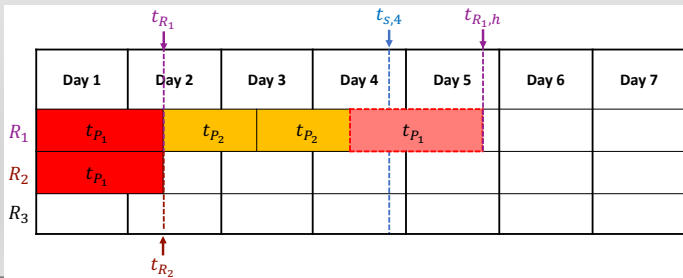
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The sales plan

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P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- ▶ 1 batch reaction for P_1 in R_1
- ▶ Tentative reactor time $t_{R_1,h} > t_{s,4} \rightarrow \text{X}$



Industrial case study

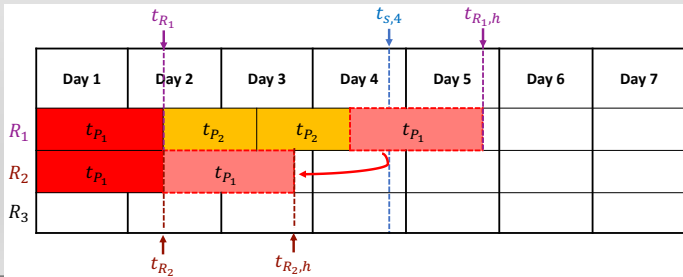
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The sales plan

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P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- ▶ 1 batch reaction for P_1 in R_1
- ▶ Tentative reactor time $t_{R_1,h} > t_{s,4} \rightarrow \text{X}$
- ▶ R_2 is enrolled
- ▶ Tentative reactor time $t_{R_2,h} < t_{s,4} \rightarrow \checkmark$





Industrial case study

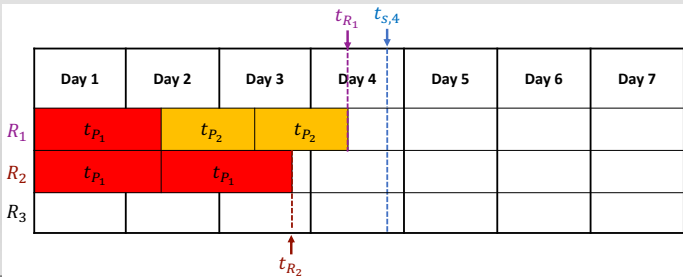
Batch Scheduling: the methodology through an example

The sales plan

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P_1	0	$S_{P_1}^{t_{s,2}}$	0	$S_{P_1}^{t_{s,4}}$	0	0	0
P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- The second sale is satisfied correctly
- $t_{R_1} = t_{P_1} + 2 * t_{P_2}$, $t_{R_2} = 2 * t_{P_1}$, $t_{R_3} = 0$





Industrial case study

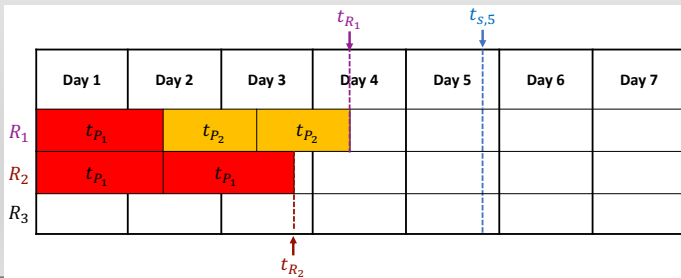
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The sales plan

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P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

- ▶ $S_{P_3}^{t_{s,5}} < W_P$
- ▶ 1 batch for P_3 required





Industrial case study

Batch Scheduling: the methodology through an example

The sales plan

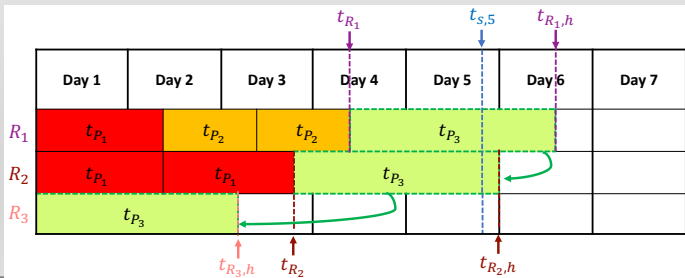
	$t_{s,1}$	$t_{s,2}$	$t_{s,3}$	$t_{s,4}$	$t_{s,5}$	$t_{s,6}$	$t_{s,7}$
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P_2	0	0	0	$S_{P_2}^{t_{s,4}}$	0	0	0
P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

► $t_{R_1,h} > t_{s,5} \rightarrow \text{X}$

► $t_{R_2,h} > t_{s,5} \rightarrow \text{X}$

► R_3 is enrolled $\rightarrow t_{R_3,h} < t_{s,5} \rightarrow \checkmark$





Industrial case study

Batch Scheduling: the methodology through an example

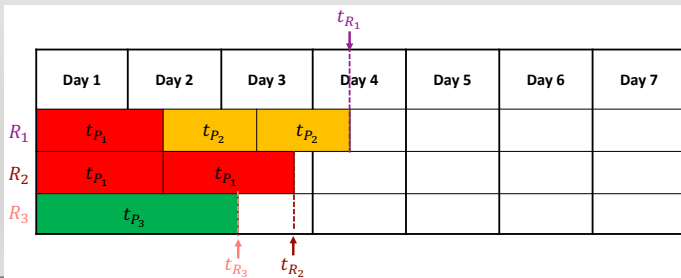
The sales plan

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P_3	0	0	0	0	$S_{P_3}^{t_{s,5}}$	0	0

Comments

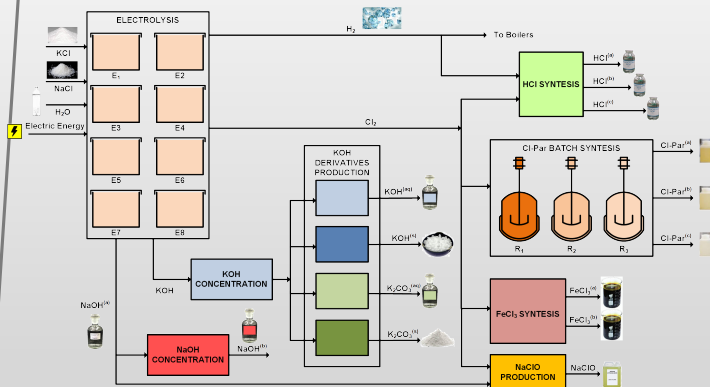
► All the sales are correctly satisfied

► $t_{R_1} = t_{P_1} + 2 * t_{P_2}$, $t_{R_2} = 2 * t_{P_1}$,
 $t_{R_3} = t_{P_3}$



Industrial case study

ALTAIR plant for chlorine derivatives



Details

16 products ($n_p = 16$)

- ▶ 12 continuous-time
- ▶ 1 non-storable and non salable (Cl_2)
- ▶ 3 batch ($n_b = 3$)

Optimization horizon

$n_h = 168 \text{ h}$

Decision variables

$n_x = (n_p - n_b)n_h = 2184$

Constraints

- ▶ 12 on stocks
- ▶ 5 safety and others

total along $n_h, > 3000$



Industrial case study

Dynamic optimization problem details

Soft and Hard constraints

► Soft:

- **2 on HCl:** $\text{HCl}^{(b)}$ can be sold after dilution to cover sales for missing $\text{HCl}^{(a)}$, $\text{HCl}^{(c)}$ can be sold after dilution to cover sales for both $\text{HCl}^{(a)}$ and $\text{HCl}^{(b)}$
- **1 on FeCl_3 :** $\text{FeCl}_3^{(b)}$ can be sold directly as $\text{FeCl}_3^{(a)}$ with a little profit loss

- **Hard:** stock bounds, electrical bounds for production of $\text{NaOH}^{(a)}$ and KOH by electrolysis

Objective Function

$$f(x) = \sigma_{\text{HCl}^{(a)}}^{n_h} + \sigma_{\text{HCl}^{(b)}}^{n_h} + \sigma_{\text{HCl}^{(c)}}^{n_h}$$

Initial condition

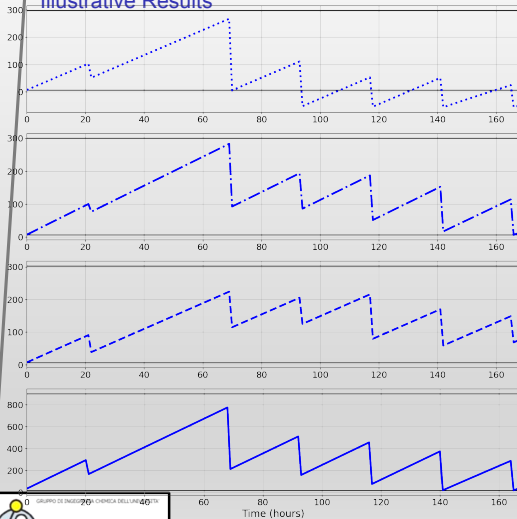
- $f(x_0) = 18$ tons
- Two types of *hard* constraints are violated

Optimal solution

- $f(x_{opt}) = 39.1$ tons
- Two types of *soft* constraints are violated

Industrial case study

Illustrative Results



$\sigma_{HCl^{(a)}}$

Stocks of $HCl^{(a)}$, $HCl^{(b)}$ and $HCl^{(c)}$

Main comments:

$\sigma_{HCl^{(b)}}$

► Stocks “zigzag” behavior is given by the sales concentrated on $t_{s,d}$

► Stock of $HCl^{(a)}$ is lower than the minimum bound (soft constraint)

$\sigma_{HCl^{(c)}}$

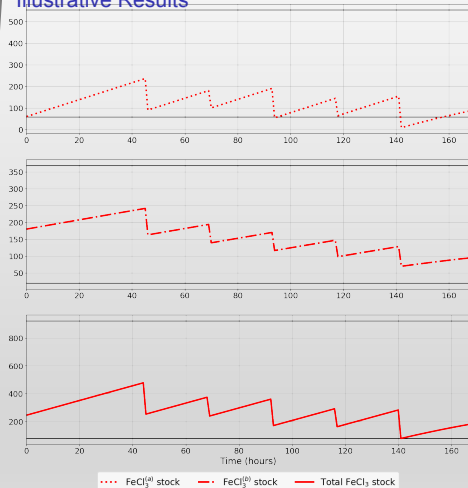
↓
↓
 $HCl^{(b)}$ or $HCl^{(c)}$ have to be diluted and sold as $HCl^{(a)}$

σ_{HCl}

► Sum of stocks of the three HCl is ok (hard constraint)

Industrial case study

Illustrative Results



$\sigma_{\text{FeCl}_3^{(a)}}$ Stocks of $\text{FeCl}_3^{(a)}$ and $\text{FeCl}_3^{(b)}$

Main comments:

► Stocks “zigzag” behavior is given by the sales concentrated on $t_{s,d}$

$\sigma_{\text{FeCl}_3^{(b)}}$ ► Stock of $\text{FeCl}_3^{(a)}$ is lower than the minimum bound (soft constraint)

↓ ↓
 $\text{FeCl}_3^{(b)}$ has to be sold as $\text{FeCl}_3^{(a)}$ with a profit loss

σ_{FeCl_3} ► Sum of stocks of the two FeCl_3 is ok (hard constraint)

Industrial case study

Conclusions

- ▶ **DRT0 algorithm** to best manage production rates based on the sales plan
- ▶ **Project** for an integrated digitalization of an industrial site according to **Industry 4.0 paradigms**
- ▶ Products continuous vs batch, storable to be sold vs consumed in real-time within the industrial site
- ▶ **Preliminary scheduling** procedure for batch productions to avoid MIP
- ▶ **A smooth implementation of a LP** to obtain always a numerically feasible solution. The scheduling procedure gives parameters used into the LP
- ▶ A post-analysis of the optimal solution gives a **feedback to the operator**
- ▶ A key instrument in a full computerization and digitalization project of the company

DRTO: Conclusions

- ▶ Direct optimizing control is a promising approach to optimization of dynamic economic performance of chemical processes
- ▶ Modeling is about making educated approximations to arrive at a model of acceptable complexity that is adequate for optimization in the presence of uncertainty
- ▶ Advances in large-scale nonlinear programming solvers and sensitivity lead to formulation of nonlinear model-based dynamic optimization that are feasibly executed within allowed time
- ▶ DRTO/EMPC scheme are likely to replace or at least complement the more conventional hierarchical RTO/LMPC architecture

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